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**Models**

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## Abstract

I construct a theory of cultural transmission in which culture acquisition takes place in two stages, first in the family where parents transmit their own culture, and later in society where children are exposed to a wider set of cultural models. The role of models is to provide information about alternatives. Cultural variants differ in how strongly they are transmitted in the family and on how attractive they are to the children's eyes. Attractiveness may depend on the actual models one can observe. I characterise the long run distribution of variants using directed trees and show that more visible cultural variants will have larger shares. Shares are also increasing in attractiveness and in family strength. When attractiveness is not context specific, variants competing with a wider set of variants, everything else equal, will have larger shares provided that copying is bidirectional. Expanding the set of models does not necessarily lead to an increase in shares.

**JEL classification:** D10, I20, J13.

**Keywords:** Cultural transmission, role models, learning.

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# 1 Introduction

Marie Curie, Zidane, Obama..., are often mentioned as role models. The *female* scientist who reached the top, the *immigrant* who made it in his new country and became a symbol of a multicultural society, the first *African American* President in US history. Characters that kick open a door in the minds of children.

“Models” are “actual” examples, their role to provide information about possibilities, to make children aware of alternatives which could be unknown until then or thought to be beyond reach. The set of “models” include not only celebrities like those mentioned above, but also peers, friends, teachers and other figures whose achievements and/or characteristics exert an influence on others’ decisions. A striking example is that of Jeetendra Prasad, the first person from Patwatoli in Bihar (India) to be admitted in an IIT (Indian Institute of Technology). He got into the IIT in 1991 and in 1997 joined PriceWaterhouseCoopers in New Jersey. Patwatoli is the home of the Patwa (weaver) community, an impoverished village which could be “expected to breed Naxalites rather than engineer graduates” (Varma (2004)). By 2002 already 22 boys from there had passed the admission exams for the IIT’s, and three times as many were looking forward to joining other engineering institutes. All came from poor families but Prasad’s success story “created a dream, and the children of poor weavers on the boondocks of Bihar were willing to chase it” (Varma (2004)). The success of El Sistema,<sup>1</sup> a music teaching program, which has rescued thousands of Venezuelan children from a likely environment of drug abuse and crime, can be explained with the same basic mechanisms.

In spite of the growing interest of economist on the process of cultural transmission there is no economic theory incorporating the role of models as carriers of information. To contribute to fill this gap I propose a theory of cultural transmission in which children learn both from their parents and from models.

I assume that children are educated first in the family, where they acquire (provisionally) a variant of a cultural trait, and later in a wider world where they learn about new possibilities. A family is characterised by its variant and by the set of models its children are exposed to. A set of models is like a menu from which children may choose, different families offering different dishes. And, as it happens with menus, the values attached to the different alternatives depends not only on the actual set of options but also on one’s family’s (culinary) culture. Cultural variants differ in two aspects: i) on how permanent or difficult to change they are and ii) on their attractiveness to the children’s eyes. Children form their own views about the worth of each of the alternatives in their set of models, assign values to them and emulate each of the models with a probability which is proportional to those values. After this second stage,

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<sup>1</sup>La Fundacion del Estado para el Sistema Nacional de las Orquestas Juveniles e Infantiles de Venezuela was founded in 1975 by Juan Antonio Abreu, economist and musician. More than two million kids have gone through the program. Participants include Gustavo Dudamel, current music director of Los Angeles Philharmonia, and many other successful musicians like Edicson Ruiz who at age 17 became the youngest ever bass player in the Berlin Philharmonia! The success of the program goes beyond star musicians. A study of the Inter- American Development Bank links participating in the program with improvements in school attendance and declines in delinquency.

culture is fixed and children become adults. My objective is to characterise the dynamics of traits.

The driving force of the dynamics is the possibility of reaching new sets of models through the process of cultural change. In particular, children, by acquiring traits which are different from those of their parents, open up different alternatives to their own (future) children, alternatives that they themselves could not even contemplate or think of when they made their choices. Consider for instance the choice of profession, and assume that there are only three alternatives (variants) to choose from: farmer, medical doctor and mathematician. Farmers are farmers from time immemorial when they live in complete isolation. But what about if farmers' children become aware of the attractive possibility to becoming medical doctors? Some children of farmers may move to the city and become doctors. Their children, by growing in a doctor's environment, will be exposed to a wider array of models which include not only doctors and (possibly) farmers but also mathematicians. The process of cultural transmission will lead to a new distribution of professions and, in the long run, some descendants of farmers will be mathematicians, some doctors and other farmers. The actual proportions will depend on the attractiveness different children attach to the different variants and on the way the sets of models are actually connected.

The main components of the model are the sets of cultural models different children are exposed to and the values different children attach to the models in their set. I assume that children's evaluations of cultural variants depend on the children's family as well as on the set of available models. For instance, the Straddle technique's attractiveness in the high jump is likely to be higher when only the Scissors-Jump is an alternative than when also the Fosbury Flop exist in the set of models.<sup>2</sup> The attractiveness of headscarfs and other religious signs, dressing styles, languages...etc, may well depend on the competing models, the same way that the attractiveness of a mobile telephone depends on the presence or not of an iPhone. Smoking maybe better seen in some circles than in others, academics' children may find more glamour in the life of a researcher than in that of a football player while children from other families may have a different rankings.

I show that the resulting dynamics can be written as a Markov chain. I can then characterise the long distribution of cultural variants in the population, provided that the process is aperiodic and irreducible. Irreducibility means, in my model, that some direct descendants of individuals with variant  $i$  will have variant  $j$ , and this holds true for any pair of variants. Since children can also be like their parents, the system is aperiodic. These two properties guarantee that the resulting dynamic system has a unique stable long run distribution which can be characterised using directed trees. More visible, more permanent and more attractive variants are shown to have larger shares in the long run. In the special case in which all children evaluate variants in the same way but are still exposed to different sets of models, the long run proportion of each variant is proportional to the total value assigned to the own set of models, and to the own merit and permanence, provided that copying is bidirectional. This condition requires that if a farmer's child can become a doctor with positive probability,

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<sup>2</sup>The model, though phrased in terms of biological parents and offsprings, allows a wider interpretation of cultural parents and cultural children, like a supervisor (belonging to a certain school of thought) and his supervisee, a coach (with his techniques) and his trainee...

then a doctor's child can become a farmer with positive, though not necessarily equal, probability. In this case if all variants happen to be equally attractive, those competing with more variants, i.e those with larger set of models, will have larger shares. Increasing the number of models does not necessarily lead to an increase in the long run share. A sufficient condition for this to be the case is that all variants be equally permanent.

The model rationalises why some groups may decide to cut off all relations with the outside world in order to avoid extinction or impose strong penalties on those who leave the community. This is optimal for communities with cultural variants which face competition from more "attractive" ones. A well known example are the Amish of North America<sup>3</sup> who advocate a simple life style with very strict codes of behaviour which looks little attractive in modern societies.<sup>4</sup> These communities are a extreme example but examples abound of groups which put enormous efforts into the preservation of their cultures when they enter in competition with others. Catalans and Basques have managed to preserve their languages, Jews, in spite of centuries of persecutions, have preserved many of their old traditions. The model can also explain how some variants may easily spread, why English is the language of science, why German and French are loosing ground as second languages in many countries, why Brazil excels in football and India in IT.

The model differs from previous models on cultural transmission in several aspects. First, all existing theoretical models (see Bisin and Verdier (2000, 2006)<sup>5</sup> and Saez-Marti and Sjögren (2008)) consider only dichotomous cultural traits, namely traits which have only two variants while I consider an arbitrary, finite number of them. This makes possible the study of an important aspect in the dynamics of culture: the introduction of new variants, be it by cultural innovation or brought in by immigration. Second, previous models have no role for role models, nor allow they for heterogeneity in the influences beyond those of the family in the first stages of life. In particular, children acquire their culture either in the family, via direct costly teaching or, if parent's efforts fail, from society. This second source of cultural influence is the same for all children, and is restricted to the parents' generation. By incorporating influences beyond the parents' cohort, I come closer to the modelling of contemporary societies where outside influences coming from other children, schools, public figures, the media... are increasingly important. Heterogeneity is introduced by allowing that children from different families be exposed to different sets of models, their choices being then constrained by their limited available opportunities.

The model presented here is not one of social networks in which agents interact with different sets of neighbours and influence each other but a model in which different people have access to different partitions of a given set of possibilities. It is the parents' culture what determines which actual partition children have access to. Acquiring a different culture

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<sup>3</sup>In 2000, according to some studies, there were 165,000 Old Order Amish in Canada and the United States. In 2006 they were 227,000. They have an average of 6.8 children per family.

<sup>4</sup>For instance private use of cars is mostly banned because it would "quicken the pace of their life, erase geographical limits, weaken social control, and eventually ruin their community." Amish children follow formal education only up to eighth grade, further education is deemed a threat that separates them from their parents, their parents' traditions and values.

<sup>5</sup>Bisin and Verdier (2006) reviews some of the most interesting applications of the theoretical model in Bisin and Verdier (2000).

implies moving to a different partition so that the offspring will have access to different choices. The same that having English rather than French as a second language may influence the holiday destination and the food one gets acquainted with and ends up eating at home, a child of a mathematician is exposed to opportunities which are different not only from those of a farmer's child but also from those of his own child if he himself decides to be a doctor. This process of jumping from one partition into another drives the inter generational transmission of culture which I analyse in the paper.

The model I propose is silent on what determines the actual partition one falls in. I do take them as given and analyse the implication for the long run distribution of culture as well as the effect of changes in the partitions and of the introduction of new variants.

The paper is organised as follows. In the next section I introduce the basic model. In section 3 I derive the general dynamics and obtain the long run distribution of cultural traits. Section 4 analyses the case with equal evaluations and bidirectional copying and study its implications for the long run distribution. Section 5 concludes.

## 2 The model

I present an overlapping generation model with a Poisson birth and death process. Time is discrete and each period is subdivided in two. At the beginning of each period the economy is populated by a large number of adults. Each adult is characterised by a fixed variant of a cultural trait. The set of possible variants is finite and denoted by  $K = \{1, 2, \dots, k\}$ , with  $k \geq 2$ . The cultural trait could be religion and the variants the possible religious denominations, it could be profession and the variants a list of alternatives including farmers, weavers, engineers... At the beginning of the first sub-period each adult has a child with probability  $\lambda$ . During this first sub-period the new born is educated in the family and through a process of learning and imitation acquires a variant of the cultural trait. Children can only acquire variants they see and since parents differ in their cultures, children from different families<sup>6</sup> end up being different.

I assume that all children acquire the family variant but some variants are pushed more forcefully than others or are more difficult to change, so that children differ in the probability of actually sticking to the family culture. Let  $p_i \in (0, 1)$  denote the probability that a child from family  $i$  does not consider revising his trait, i.e.; will be like his parents. Let  $\mathbf{p}$  be the  $k$ -dimensional vector which has  $p_i$  as its  $i$ -th element. This vector summarises parents' first sub-period influence.

In the second sub-period children are exposed to new models and can decide to revise the family trait. Let  $M \subseteq K$  denote a set of models and let  $f_i(M)$  be the probability that a child who revises his trait and faces models in  $M$  copies variant  $i$ . We assume that  $f_i(M) \geq 0$  whenever  $i \in M$  and  $f_i(M) = 0$  when  $i \notin M$  so that children can not acquire variants they are not aware of.

The set of models  $M \subseteq K$  includes peers but also other individuals like teachers, neighbours, friends..., and people one is in no direct contact with but of whose existence and

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<sup>6</sup>Since a family consists of a parent and a child, I use the terms parent and family interchangeably.

achievements is aware of and can relate to. The child of a farmer in a remote village in the Swiss Alps may not be aware of the existence of mathematicians or opera singers but may well know of the existence of medical doctors and vets. The set of models is allowed to vary across children from different families. After this second learning/imitation stage, culture is definitively fixed and children are adults with a fixed culture which will determine the set of models their own children will have access to.

In order to keep the population constant we assume, following Hauk and Saez Marti (2002), that at the end of the period, (old) adults die with probability  $\lambda$ . The state of the economy is described by the  $k$ -dimensional vector  $\mathbf{x}(t)$  indicating the proportion of adults having each of the variants at the beginning of period  $t$ .

I look for the steady state of the following system of difference equations describing the dynamics of the population distribution of cultural variants:

$$x_i(t+1) = (1-\lambda)x_i(t) + \lambda(p_i x_i(t) + \sum_{j=1}^k f_i(M^j)(1-p_j)x_j(t)) \quad i = 1, 2, \dots, k \quad (1)$$

where  $M^j \subseteq K$  is the set of models of a  $j$ -child. The first term on the RHS are the surviving adults, the second are those children who acquired variant  $i$  and did not change it, and the last term are those who revised their culture and acquired variant  $i$ .

In order to close the model I need to characterise children's choice of variants given their set of cultural models. I assume that children attach some merit or attractiveness to each of the models in their set of models. The evaluations may depend on the actual set the child can observe and on the family one comes from. For instance the value of learning a foreign language may depend on the existence or not of foreigners in one's environment. The value given to Catalan or Basque relative to Castilian may depend on whether one's parents are immigrant from other Spanish communities.

Let  $a_j^i \geq 0$  be the attractiveness or merit of variant  $j$  for an  $i$ -child who faces models in group  $M^i$  and assume that  $a_j^i = 0$  for all  $j \notin M^i$  so that children cannot assign values to things of which existence they are not aware. Given set  $M^i$ , the probability of copying trait  $j$  is,

$$f_j(M^i) = \frac{a_j^i}{\sum_{k \in K} a_k^i} = \frac{a_j^i}{\sum_{k \in M^i} a_k^i}, \quad (2)$$

so that more attractive variants are copied with higher probability.

In the next section I analyse the dynamics (1) in the most general case in which children from different families may be confronted with different sets of models and may evaluate traits differently. In a later section I will analyse the special case in which all children evaluate traits in the same way.

### 3 The dynamics

The system of difference equations (1) can be written as

$$\mathbf{x}(t+1) = \mathbf{x}(t)\mathbf{\Gamma}, \quad (3)$$

where  $\mathbf{\Gamma} = [\gamma_{ij}]$  is a  $(k \times k)$  matrix with entries

$$\gamma_{ij} = \begin{cases} \lambda(1-p_i)f_j(M^i) & i \neq j \\ (1-\lambda) + \lambda(p_i + (1-p_i)f_i(M^i)) & j = i, \end{cases} \quad (4)$$

and  $f$  is given by (2).

Since  $\mathbf{\Gamma}$  is a stochastic matrix (all entries are non negative and all its row sums are unity) our deterministic dynamic system is mathematically equivalent to a Markov chain and I can apply standard results to characterise the long run distribution of variants in the population. In particular if  $\mathbf{\Gamma}$  is such that the process is aperiodic and irreducible there exist a unique stationary distribution  $\mathbf{x}^*$  so that,

$$\mathbf{x}^* = \mathbf{x}^*\mathbf{\Gamma} \quad (5)$$

moreover,

$$\lim_{t \rightarrow \infty} \mathbf{x}(t) = \mathbf{x}^* \quad (6)$$

for any initial distribution.

Aperiodicity is guaranteed by the fact that  $\gamma_{ii} > 0$ , namely children are like their parents with positive probability. Irreducibility means in our context that some descendants of  $i$ -individuals are  $j$ -individuals after a finite number of generations. And this is true for any  $i$  and any  $j$ . Consider for instance our example with the three professions: medical doctors, mathematicians and farmers, to be numbered 1, 2 and 3, respectively. Assume that  $M^1 = \{1, 2, 3\}$ ,  $M^2 = \{1, 2\}$  and  $M^3 = \{1, 3\}$ , so that all children know about the profession of their parents, but only the doctors' children are aware of all other professions. Farmers' children know about doctors and so do the mathematicians' but they ignore each other. The resulting system is irreducible because some children of the mathematicians become doctors and since some children of doctors become farmers, some mathematicians will have farmers as grandparents. Similarly some grandchildren of farmers will be mathematicians because their parents were doctors and some of the doctors' parents are farmers. If instead  $M^3 = \{3\}$ , so that farmers' children think being farmers is their only alternative, the system is reducible and farmers are farmers and will always be farmers.

Proposition 1 characterises the unique long run distribution of variants and shows that it depends on the probability of revision, on the total attractiveness of the peer group and on how connected these groups are. I use a Lemma from Freidlin and Wentzel (1984, lemma 3.1) which uses a particular type of directed graphs,  $z$ -trees, to characterised the long run distribution of aperiodic irreducible finite state Markov chains. Intuitively a  $z$ -tree indicates how a state  $z \in K$  of a finite Markov chain (a variant in our application) can be reached from any other state without passing through any state more than once. Formally, a  $z$ -tree is a collection of arrows between elements of  $K$  such that i) every element  $i \in K \setminus \{z\}$  is the origin



of one and only one arrow that leads to some other state  $j \in K$ , ii) there is a unique path starting in  $i$  that leads to  $z$ , and iii) there are no close loops. Figures 1 shows all possible 3-trees (farmer-trees) for the three-variant example.

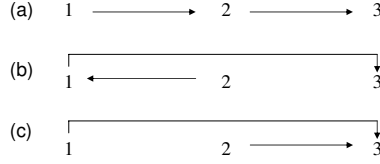


Figure 1: All possible 3-trees

Freidlin and Wentzel (1984) associate to the arrow linking  $i$  with  $j$ , to be denoted  $(i \rightarrow j)$ , the probability  $\gamma_{ij}$  that the transition occurs, to each  $z$ -tree the product of the probabilities of all its arrows and to each state  $z \in K$  the sum of all the numbers assigned to all its  $z$ -trees. Let  $q_z$  be the resulting number,

$$q_z = \sum_{h \in T_z} \prod_{i \rightarrow j \in h} \gamma_{ij} \quad (7)$$

where  $T_z$  is the set of all  $z$ -trees and  $h$  is a  $z$ -tree. Note that some elements in the sum above may be zero since some of the  $\gamma_{ij}$ 's could be zero. If the process is irreducible  $q_z$  is positive since any state can be reached from any other in a finite number of steps.

In the 3 variant case (see Figure 1)  $q_3 = \gamma_{12}\gamma_{23} + \gamma_{21}\gamma_{13} + \gamma_{13}\gamma_{23}$ , where  $\gamma_{ij}$  is the probability that  $j$  is reached from  $i$  in one step. In the three profession example the only 3-tree with a positive value is (b) since  $1 \in M^2$  and  $3 \in M^1$  so that both  $\gamma_{21}$  and  $\gamma_{13}$  are positive. The other two trees, (a) and (c), have an arrow linking 2 and 3, and this direct transition happens with probability 0 since the son of a mathematician cannot become a farmer ( $3 \notin M^2$ ) and hence  $\gamma_{23} = 0$ . In this case  $q_3 = \gamma_{21}\gamma_{13}$ .

Aperiodicity and irreducibility guarantee the existence of a unique stationary distribution  $\mathbf{x}^*$ . Let  $\mathbf{q}$  be the  $k$ -dimensional vector which has  $q_i$  as its  $i$ -th element. Freidlin and Wentzel (1984, chapter 6, Lemma 3.1) prove the following,

**LEMMA 1.** *The vector  $\mathbf{x}^*$  is proportional to  $\mathbf{q}$ .*

I can now state my main result:

**PROPOSITION 1.** *Assume (2) and that the resulting Markov process is irreducible. Then,*

$$x_i^* = \frac{(\sum_{h \in T_i} \prod_{k \rightarrow j \in h} a_j^k)(1 - p_i)^{-1}(\sum_{k \in M^i} a_k^i)}{\sum_{j=1}^k (\sum_{h \in T_j} \prod_{k \rightarrow s \in h} a_s^k)(1 - p_j)^{-1}(\sum_{k \in M^j} a_k^j)} \quad i = 1, 2, \dots, k.. \quad (8)$$

*Proof.* It follows from Freidlin and Wentzell (1984) that

$$x_i^* = \frac{q_i}{\sum_{k \in K} q_k}. \quad (9)$$

Substituting  $\gamma_{ij}$  in (7) we obtain that

$$q_z = \frac{\prod_{k \neq z} (1 - p_k)}{\prod_{k \neq z} (\sum_{i \in M^k} a_i^k)} \sum_{h \in T_z} \prod_{i \rightarrow j \in h} a_j^i. \quad (10)$$

Plugging (10) in (9) and multiplying and dividing by  $\prod_{k \in K} (1 - p_j)^{-1} (\prod_{k \in K} \sum_{i \in M^j} a_k^j)$  delivers (8). Q.E.D.

The ratio

$$\frac{\sum_{h \in T_i} \prod_{k \rightarrow j \in h} a_j^k}{1 - p_i} \quad (11)$$

captures the fitness of the variant. This depends on i) how attractive, from the point of view of the potential adopters, and how connected it is and on ii) how easily it can be changed.

Since all  $i$ -trees end in variant  $i$ , it has to be the case that each of the terms in the numerator contain at least one  $a_i^j$  for  $j \neq i$ , so that variants which look very unattractive to those considering its adoption, will have lower shares. Increases in the attractiveness of the own variant, as well as decreases in the probability revising the variant, increase unambiguously the long run shares.

Continuing with the example developed above, consider the example of the isolated community of farmers ( $M^3 = \{3\}$ ) whose children learn, at a certain point in time, about the attractive possibility to becoming medical doctors. Their set of models becomes larger ( $M^3 = \{1, 3\}$ ) and some farmers' children become doctors. Since doctors have access to a larger world,  $M^1 = \{1, 2, 3\}$ , and the prospect of becoming a mathematician is also attractive for them, some of the doctors' children become mathematicians, others farmers and some keep on being doctors. Some mathematicians' children become also doctors since  $M^2 = \{1, 2\}$ . After some time a new stationary distribution is reached with mathematicians, doctors and farmers, all having farmers in their family trees. Figure 2 below illustrates the dynamics.

Initially all are farmers (circles). At  $t = 2$  their set of models become larger and some farmer's children become active doctors at  $t = 3$  (stars). At  $t = 4$  some doctors' children become mathematicians (diamonds). Note that in this example it is not the variant which is more connected the one which has a larger long-run share but the one which is most preferred (mathematicians) by those who are more connected (the doctors).

I have not ruled out the possibility that children, when deciding to change their culture consider also their own variant. For this to be the case  $a_i^i$  should be positive. If some children react against their family and want to be different then  $a_i^i = 0$  for those children, and the long run shares as given by (8) would be smaller.

I analyse next the case in which all children evaluate variants in the same way.

## 4 Equal evaluations

Assume now that all children evaluate the variants equally, so that the merits are independent of the actual sets of models, namely  $a_i^j = a_i$  for all  $j$ . The following proposition shows that when a simple structure is introduced in the way the sets of models are connected, the actual

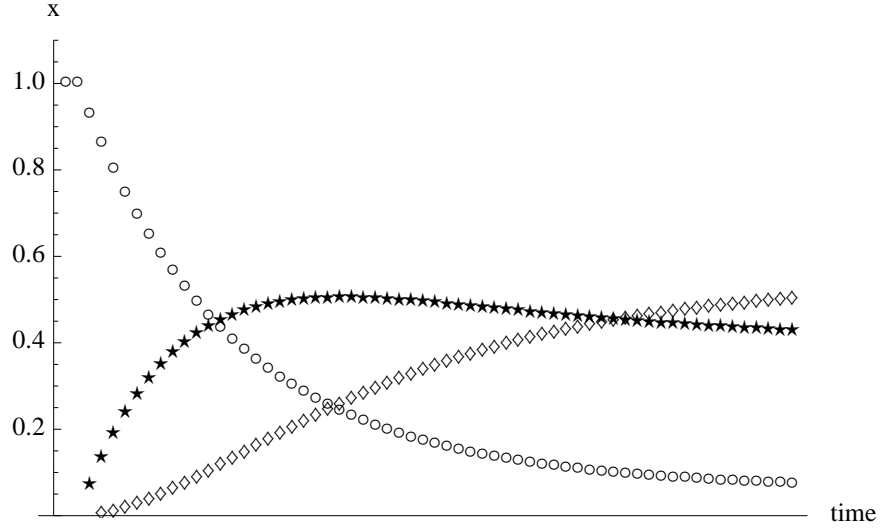


Figure 2: My ancestors were farmers.

connections cease to be important. The condition requires copying to be bi-directional so that if a child of type  $i$  can acquire trait  $j$ , then a  $j$ -child can acquire trait  $i$ . This assumption is reasonable when the set of models are the peers one interact with, so that observation is bidirectional. Schooling and interactions may help homogenise the evaluations of existing traits. In the proposition I still allow for differences in the set of models.

**PROPOSITION 2.** Assume  $a_i^j = a_i$  for all  $j$ ,  $i \in M^j$  if and only if  $j \in M^i$ , and that the process defined by (3) is irreducible, then

$$x_i^* = \frac{a_i(1 - p_i)^{-1} \sum_{k \in M^i} a_k}{\sum_{j=1}^k a_j(1 - p_j)^{-1} \sum_{k \in M^j} a_k} \quad i = 1, 2, \dots, k. \quad (12)$$

*Proof.* We show first that  $x_i^* p_i + \sum_j x_j^* (1 - p_j) f_i(M^j) = x_i^*$ ,

$$\begin{aligned}
& \frac{a_i(1 - p_i)^{-1} \sum_{k \in M^i} a_k}{\sum_{j=1}^k a_j(1 - p_j)^{-1} \sum_{k \in M^j} a_k} p_i \\
& + \sum_{l \in M^i} \frac{a_l(1 - p_l)^{-1} \sum_{k \in M^l} a_k}{\sum_{j=1}^k a_j(1 - p_j)^{-1} \sum_{k \in M^j} a_k} \frac{(1 - p_l) a_i}{\sum_{k \in M^l} a_k} \\
& = \frac{a_i(1 - p_i)^{-1} \sum_{k \in M^i} a_k}{\sum_{j=1}^k a_j(1 - p_j)^{-1} \sum_{k \in M^j} a_k} p_i \\
& + \sum_{l \in M^i} \frac{a_l a_i}{\sum_{j=1}^k a_j(1 - p_j)^{-1} \sum_{k \in M^j} a_k} \\
& = \frac{a_i(1 - p_i)^{-1} \sum_{k \in M^i} a_k}{\sum_{j=1}^k a_j(1 - p_j)^{-1} \sum_{k \in M^j} a_k}.
\end{aligned}$$

Substituting  $x_i(t)$  in (1) by  $x_i^*$  gives  $x_i(t + 1) = x_i^*$ . Uniqueness follows from the ergodicity of the Markov chain. Q.E.D.

Proposition 2 is a particular case of Proposition 1. Two restrictions have been introduced: i) all children evaluate variants in the same way and ii) imitation has to be bidirectional. None of these assumptions was needed to obtain the long run distribution characterised in Proposition 1. Curiously enough these two assumptions imply that the long run distribution does not depend on the actual set of directed trees, as can be seen by comparing (12) and (8).

Since the long run distribution is unique, I can, by some reverse engineering, obtain an expression for the vector  $\mathbf{q}$  (see equation (7)). Direct comparison of (12) and (8) leads to

$$\sum_{h \in T_i} \prod_{j \rightarrow s \in h} a_s^j = a_i Q(T_i) \quad i = 1, 2, \dots, k, \quad (13)$$

where  $Q(T_i)$  is a constant which depends on the set of  $i$ -trees. It has to be necessarily the case that  $Q(T_i) = Q(T)$  for all  $i$ , since  $Q(T_i)$  cancels out when ratios are taken to obtain  $x_i$  using  $\mathbf{q}$  (see equation (9)). I can now write the expression for  $q_i$

$$q_i = a_i Q(T) \frac{\prod_{k \neq i} (1 - p_k)}{\prod_{k \neq i} (\sum_{s \in M^k} a_s)} \quad i = 1, 2, \dots, k. \quad (14)$$

Note that, by construction, changes in  $M^i$  do not have any effect on  $q_i$ , provided that all other sets of models remain unchanged. Expanding  $M^i$  will have an effect on all other  $q$ 's, by increasing the number of possible directed trees and changing the probabilities of the transitions. The share of  $i$  will fall in equilibrium since we open a path out of  $i$  without opening one heading towards  $i$ . The same is not true if changes in  $M^i$  are accompanied by changes in other sets. The result of Proposition 2 is obtained under the assumption that if one element, let us say  $j$ , is added to  $M^i$ , then the element  $i$  has to be added to  $M^j$ . By

expanding  $M^i$  we increase the probability that a child of an  $i$ -parent is not like his parent, but at the same time we make positive the probability that the children of  $j$ -parents become  $i$ , and change the probabilities that they acquire any of the other traits in  $M^j$ . It happens that these changes affect all the  $q$ 's in the same manner, so that the effect on the long run distribution of the new constant  $Q(T')$  will, again, cancel out. All the effect on the long run distribution comes from the changes in the total values of the sets  $M^i$  and  $M^j$ , which jointly with the  $p$ 's affect the probabilities of transition.

The following corollary gives the condition needed for an expansion of the set of models to result in the increase of the long run share of a variant. Assume that initially  $i \notin M^j$  and  $j \notin M^i$  and that the economy is in its long run equilibrium  $x_i^*$ . In a certain moment in time  $M^i$  expands to include  $j$  (and as a result  $M^j$  to include  $i$ ). The new sets to be considered are  $\hat{M}^i = M^i \cup \{j\}$ ,  $\hat{M}^j = M^j \cup \{i\}$  and  $\hat{M}^s = M^s$  for all  $s \neq i, j$ . We show that heterogeneity in the  $p$ 's is needed in order to have the possibility that the equilibrium share of  $i$  decreases.

**COROLLARY 1.** *Assume that the conditions of proposition 2 hold. Including element  $j$  in  $M^i$  and  $i$  in  $M^j$  increases the equilibrium share of  $i$  when*

$$\sum_{s \in K \setminus \{i, j\}} \frac{a_s \sum_{k \in M^s} a_k}{1 - p_s} > \frac{a_i \sum_{k \in M^i} a_k - a_j \sum_{k \in M^j} a_k}{1 - p_j} \quad (15)$$

*This condition always holds true if  $p_s = p$  for all  $s \in K$ .*

*Proof.* We denote by  $x^*$  and  $\hat{x}^*$  the corresponding long run shares as given by (12),

$$\hat{x}_i^* = \frac{\frac{a_i}{1-p_i} \sum_{k \in \hat{M}^i} a_k}{\sum_{s \in K} \frac{a_s}{1-p_s} \sum_{k \in \hat{M}^s} a_k} = \frac{\frac{a_i}{1-p_i} \sum_{k \in M^i} a_k + \frac{a_i a_j}{1-p_i}}{\sum_{s \in K} \frac{a_s}{1-p_s} \sum_{k \in M^s} a_k + \frac{a_i a_j}{1-p_i} + \frac{a_i a_j}{1-p_j}} \quad (16)$$

$$x_i^* = \frac{\frac{a_i}{1-p_i} \sum_{k \in M^i} a_k}{\sum_{s \in K} \frac{a_s}{1-p_s} \sum_{k \in M^s} a_k}. \quad (17)$$

$\hat{x}_i^* - x_i^*$  is positive whenever

$$\frac{a_i a_j}{1 - p_i} \sum_{s \in K} \frac{a_s}{1 - p_s} \sum_{k \in M^s} a_k > \left( \frac{a_i}{1 - p_i} \sum_{k \in M^i} a_k \right) \left( \frac{a_i a_j}{1 - p_i} + \frac{a_i a_j}{1 - p_j} \right). \quad (18)$$

Rearranging (18) we obtain (15).

When  $p_s = p$  for all  $s$  (15) simplifies to

$$\sum_{s \in K \setminus \{i\}} a_s \sum_{k \in M^s} a_k - a_i \sum_{k \in M^i} a_k. \quad (19)$$

Since each of the elements of the second term has a identical counterpart in the first term (by the bidirectional assumption) (19) is always positive.

Q.E.D.

To understand this result it is useful to see which conditions other than the heterogeneity in the elements of  $\mathbf{p}$ , lead to a decrease in  $x_i^*$ . A necessary condition for (15) to be violated is that

$$a_i \sum_{k \in M^i} a_k > a_j \sum_{k \in M^j} a_k. \quad (20)$$

This will be the case if  $j$  and/or the models in its set are few and unattractive relative to variant  $i$  and the models in its set. If condition (20) is satisfied, (15) will be more easily violated if  $p_j$  is large.

It is worth noting that: i) at least one of the two variants which open a new contact will increase its share and ii) all other variants will experience a decline in their equilibrium shares. It can easily be seen from (15) that if  $i$  decreases its share, then the share of  $j$  has to increase, it is possible, though, that both increase. Since  $\hat{M}^s = M^s$  for all  $s \neq i, j$  it is always the case that the share of  $s$  decreases,

$$\hat{x}_s^* = \frac{\frac{a_s}{1-p_s} \sum_{k \in M^s} a_k}{\sum_{g \in K} \frac{a_g}{1-p_g} \sum_{k \in M^g} a_k + \frac{a_i a_j}{1-p_i} + \frac{a_i a_j}{1-p_j}} < \frac{\frac{a_s}{1-p_s} \sum_{k \in M^s} a_k}{\sum_{g \in K} \frac{a_g}{1-p_g} \sum_{k \in M^g} a_k} = x_s^*. \quad (21)$$

Opening up a contact with unattractive, relatively isolated and difficult to change variants may lead to a decline in the equilibrium value of all the variants but the unattractive one. This could explain why unattractive variants like drug use, or membership in some (religious) sects do not disappear but increase their shares when entering in contact with other groups. The few new adepts are locked in.

To isolate further the role of  $p$ , assume that  $a_s = a$  for all  $s$ , so that we can abstract from differences in attractiveness. In this case

$$x_i^* = \frac{n_i(1-p_i)^{-1}}{\sum_{j \in K} n_j(1-p_j)^{-1}} \quad i = 1, 2, \dots, k, \quad (22)$$

where  $n_j$  denotes the number of elements in  $M^j$ ,  $j = 1, 2, \dots, k$ . Condition (15) simplifies to

$$\sum_{s \in K \setminus \{i, j\}} \frac{n_s}{1-p_s} > \frac{n_i - n_j}{1-p_j}. \quad (23)$$

so that the share of  $i$  will always increase whenever  $n_j \geq n_i$  or, if this is not the case, when  $p_j$  is small enough.

I finally characterise the long run distribution when all sets of models are identical. This could be the case if all children attend similar schools and have access to the same information about possibilities.

**COROLLARY 2.** *Assume that the conditions of proposition 2 hold, and that  $M^i = K$  for all  $i$ , then*

$$x_i^* = \frac{a_i(1-p_i)^{-1}}{\sum_{j \in M} a_j(1-p_j)^{-1}} \quad i = 1, 2, \dots, k. \quad (24)$$

Once differences in information about the available opportunities disappear, those variants which have the higher ratio

$$\frac{a_i}{1 - p_i} \tag{25}$$

will have larger shares in the long run. High ratios are associated with variants which are very permanent and/or very attractive. This can explain fashions, easy to change and easily outmoded by a new variant coming along every six months, as well as the survival of bad habits which though unattractive are difficult to change.

## 5 Conclusion

Through the paper I have assumed that the valuations given to the different variants depend on which variants are actually available. The theory predicts that innovations which render all existing ones inferior and eventually useless, will take over the whole population. The channel through which cultural innovations spread are young agents who are exposed to the novelty and for whom it is neither too late nor too difficult to change. Consider for instance the Fosbury Flop, a cultural innovation that changed the high jump for ever at the 1968 Mexico City Olympic Games. In those Games Fosbury, the only one using the technique, established a new world record and won the gold medal. In the following games (Munich 1972) already 28 out of 40 competitors used the four year old technique. Four years later, in Moscow 1980, 13 out of 16 finalists jumped using the flop. Nowadays it is the standard.

As the Fosbury example illustrates, changes in the relative attractiveness of variants may have long lasting effects. Those changes are not necessarily brought about by the introduction of a new variant but can be the result of policy interventions or the product of new available information. An obvious example is smoking where public awareness of its dangers has had dramatic effects on its attractiveness and consequently on its spread. Yet, the awareness of the risks associated with smoking differs across groups and so do smoking rates. Less obvious examples include information campaigns aiming at changing attitudes. As Hauk and Saez Marti (2002) document, part of the success of the anti-corruption campaign in Hong-Kong in the 60's and 70's can be attributed to the enormous efforts which were devoted to changing children's views about corruption by exposing them to new and better models they could feel identified with (see Klitgaard (1988)).

The model presented here is a statistical model in which parents and children behaviour is captured by some fixed parameters. I take as a primitive that some traits are pushed more strongly than others or are more difficult to change, that children differ in the amount of information they have about alternatives, and that their choice is driven by some evaluation guided by the comparison of the currently available models. This model is a first step to understanding the dynamics of culture and the importance of models.

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